☺ Chapter 4 Notes ☺

4.8 – Function Notation

Daily Objectives:

1. Define composition of functions and learn the notation.
2. See transformations of two or three steps as the composition of functions.
3. Apply composition to real-world contexts.
4. Distinguish composition from the product of functions.
5. Understand composition both graphically and numerically.

COMPOSITION OF FUNCTIONS: A FINANCIAL EXAMPLE

The composition of functions is an important topic. It is often helpful to think of a function as a rule. The composition of functions consists of applying one rule, getting a result, and then applying the second rule to what you obtained from the first rule.

PREMISE:

A store selling very expensive items will, on February 8, sell any item for $50 less than the price. On any day in February, the store will give a discount of 15% to any customer who can prove that he/she contributed to a local charity.

OK, let x be the price of an item in the store.

If $P(x)$ is the price you will pay for an item on Feb. 8, then $P(x) = x - 50$.

If $D(x)$ is a price discounted at 15% , then, in February, the amount you will pay is $D(x) = 0.85x$

This should make sense, folks. You have *MATH POWER*. You have two functions, P and D, which depend on the value of x, the price. If you don't know x, you can't calculate $P(x)$ or $D(x)$. And, note that if you do know x and "follow the rules," you get one value for $P(x)$, and one value for $D(x)$. That is, P and D are truly functions.

P is a rule that says "take the price, then subtract $50."
D is a rule that says "take the price, then take 85% of it."

Now, on February 8, my buddy, Behringer, who has a receipt to prove he has donated to a local charity, plans to make a purchase in this store. Behringer, who does have MATH POWER, realizes that it may make a difference if the clerk applies the $50 discount first and then the 15% discount or applies the 15% discount and then the $50 discount.

Recall
$P(x) = x - 50$
$D(x) = 0.85x$ (Why do we multiply the pirce by .85?)

OK, folks... WHAT IS Behringer thinking about here? Discuss with a partner and determine the final cost for a $150 item.

150 - 50 = 100 × .85 = $85   150 × .85 = 127.50 - 50 = $77.50

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What we were doing is called Composition of functions.

> **composition of functions** The process of using the output of one function as the input of another function. The composition of f and g is written $f(g(x))$. (237)

Let's do some general computations. Remember, $P(x) = x - 50$ and $D(x) = 0.85x$.

$P(D(x)) =$ $.85x - 50$

$D(P(x)) =$ $.85(x-50) = .85x - 42.5$

If Mr. Behringer had a choice of order, which would he prefer? Does this agree with your result from the $150 item? Taking the 15% discount first gives the biggest discount.

You can, I hope, look at this and realize that there is a difference of $7.50, regardless of the price. In other words, the order in which the rules are applied does make a difference. OK, let's accept the fact that we are probably not going to have a heart attack over a difference $7.50, but that misses the point entirely. This is just one simple problem. In the real world, decisions are made in which the order things are done can make a difference of hundreds, thousands, or perhaps millions of dollars.

In this simple problem, if a customer enters the store on Feb. 8 with evidence that he/she has donated to a local charity, which is better? $P(D(x))$ or $D(P(x))$? You realize, of course, that this depends upon who is asking the question... the customer or the store manager!

The store manager would, of course, prefer $D(P(x))$. If the rules were applied in this order, then the composition rule $D(P(x))$ could be replaced by a simple rule, say W, that says
$D(P(x)) = W(x) = 0.85x - 42.50$.
In other words, the simplified rule W would say "take 85% of the listed price, then subtract $42.50."

Also, from the standpoint of the store, suppose on Feb. 8, one thousand items are purchased by people who can prove they donated to a local charity. Can you use your MATH POWER and determine that the order of the rules makes a difference of $7.50x(1,000) = $7,500 to the store?

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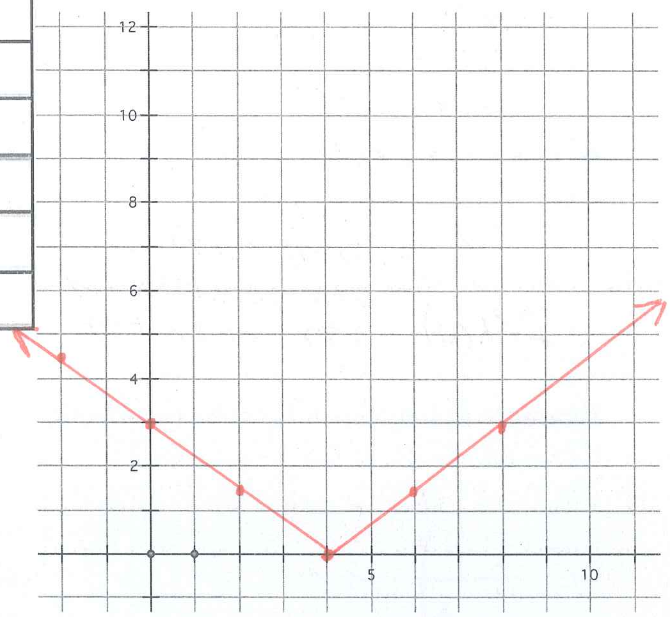
Example A: Consider the functions $f(x)=\frac{3}{4}x-3$ and $g(x)=|x|$. What will the graph of $y=g(f(x))$ look like?

x	f(x)
-2	$-4\frac{1}{2}$
0	-3
2	$-1\frac{1}{2}$
4	0
6	$1\frac{1}{2}$
8	3

f(x)	g(f(x))
$-4\frac{1}{2}$	$4\frac{1}{2}$
-3	3
$-1\frac{1}{2}$	$1\frac{1}{2}$
0	0
$1\frac{1}{2}$	$1\frac{1}{2}$
3	3



x	g(f(x))
-2	$4\frac{1}{2}$
0	3
2	$1\frac{1}{2}$
4	0
6	$1\frac{1}{2}$
8	3

The functions f and g are defined by sets of input and output values.

$f=\{(5, 0), (-1, 1), (-3, 4), (1, 2), (3, 4), (-2, 6)\}$

$g=\{(4, -1), (0, -2), (1, -1), (2, -2), (6, 0)\}$

a. What is the domain of f?

$\{-3, -2, -1, 1, 3, 5\}$

b. What is the range of g?

$\{-2, -1, 0\}$

c. Find $f(g(4))$.

$g(4)=-1$

$f(g(4))=$

$f(-1)=\boxed{1}$

d. Find $g(f(-3))$.

$g(f(-3))$

$g(4)=\boxed{-1}$

$f(-3)=4$

e. Find $f(g(f(5)))$.

$f(5)=0$

$f(g(0))$

$f(-2)=\boxed{6}$

$g(0)=-2$

f. Find $g(f(g(0)))$.

$g(0)=-2$

$g(f(-2))$

$g(6)=\boxed{0}$

$f(-2)=6$

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Example 3: Given $f(x)=(x-3)^2$ and $h(x)=8x+10$, find:

a. $f(h(4))$

$h(4) = 8(4)+10$
$= 32+10$
$h(4) = 42$
$f(h(4)) = f(42) = (42-3)^2$
$= 39^2$
$f(h(4)) = 1521$

b. $h(f(11))$

$f(11) = (11-3)^2$
$= 8^2$
$f(11) = 64$
$h(f(11)) = h(64) = 8(64)+10$
$= 512+10$
$h(f(11)) = 522$

c. $f(f(1))$

$f(1) = (1-3)^2$
$= (-2)^2$
$= 4$
$f(f(1)) = f(4) = (4-3)^2$
$= 1^2$
$f(f(1)) = 1$

Example 4: Given: $f(x)=3x-7$ and $g(x)=12-x$, find:

a. $f(g(x))$

$f(g(x)) = 3(12-x)-7$
$= 36-3x-7$
$f(g(x)) = 29-3x$

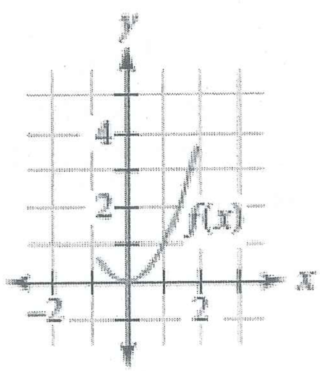
b. $g(f(x))$

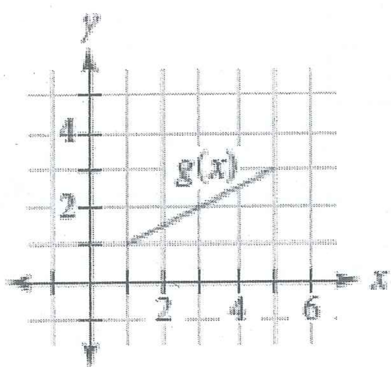
$g(f(x)) = 12-(3x-7)$
$= 12-3x+7$
$g(f(x)) = 19-3x$

c. $f(f(x))$

$f(f(x)) = 3(3x-7)-7$
$= 9x-21-7$
$f(f(x)) = 9x-28$

Example 4: Let $f(x)$ and $g(x)$ be the functions graphed below. What is the domain of $f(g(x))$?





Domain of $g(x) \Rightarrow 1 \le x \le 5$

Becomes

Range of $g(x) \Rightarrow 1 \le g(x) \le 3$

Domain $f(x) \Rightarrow 1 \le x \le 3$

but only values $1 \le x \le 2$ are actually in domain of $f(x)$.

The x-values that produced $1 \le g(x) \le 2$ are $1 \le x \le 3$.